THE RIEMANN Ξ-FUNCTION UNDER REPEATED DIFFERENTIATION

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Abstract. Differentiation causes the small gaps between zeros of a given real entire function with order 1 to become larger and the larger gaps to become smaller. I show that for the Riemann Ξ-function there exists $\langle A_n \rangle$ and $\langle C_n \rangle$ with $C_n \to 0$ such that

$$\lim_{n\to\infty} A_n \Xi^{(2n)}(C_n z) = \cos z$$

uniformly on compact subsets of $\mathbb{C}$. With the method one can prove the same result for the analogues of the Riemann Ξ-function from automorphic L-functions. Also I apply the method to the Fourier transform of $\varphi(t)$ where $\varphi(t)$ is even, continuous from $\mathbb{R}$ to $\mathbb{R}$, and $\varphi(t) = Q(t)e^{P(t)}e^{bt}(1+o(1))\ (t\to\infty)$ such that b is real, $P(t)$, $Q(t)(\not\equiv 0)$ are even polynomials with real coefficients, and the leading coefficient of $P(t)$ is negative. For these Fourier transforms I have the similar results as the Riemann Ξ-function.

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